Dynamical Systems - Math 3280
Mathematica: From Algebra to Dynamical Systems© - Version 13

Edit your document (Remove extras and errors. But, ensure it works correctly). If needed, add comments. It is not necessary to respond to questions inside the parentheses.

1. You may access the required software, called Mathematica, in the following three ways. 1. Get your own free copy at https://www.weber.edu/software/mathematica_request.html . 2. Any campus computer lab, including Tracy Hall Computer Lab, TY 101R, and Elizabeth Hall Computer Lab, EH 214. 3. Virtual Lab: See http://www.weber.edu/virtuallab .

1. Here is how to get on Mathematica:

   Log in on a computer. Move the mouse pointer to the **Mathematica icon**, which looks like the above star like symbol, and rapidly **click twice** using the **left mouse button**. You may find the mathematica icon on the desktop, under the button Start - Programs - Wolfram or Mathematica, or by doing a search for Mathematica. Then click on NewDocument , or on New Notebook, if a notebook doesn't automatically open. If you need help, ask the lab aide.

2. Entering information is like using a word processor. You can also use the arrow and backspace keys for editing. Try typing some text and practice editing it. Mathematica displays all work in "cells". The brackets on the far right indicate the cell. You can create a new cell for your next input by clicking the mouse below the cell you have been working in. First a horizontal line will appear and then when you type a new cell will be created.

   Throughout this lab and future labs input the statements in `True Type` font exactly as is!

3. Mathematica will do arithmetic operations using the usual symbols $+, -, *, /, \hat{ }$. A blank space between two terms also indicates multiplication but using $*$ might facilitate later editing. To see how it works, enter the following calculation followed by the appropriate **Evaluation** (Execution) key command; the **Enter** button on the number pad (not $\boxed{\longleftarrow}$ on the main keyboard) should work on all computers.

   $$4*8 \quad \boxed{\textbf{Enter}} \text{ (in the number pad, not the Return key)}$$

   (In place of the $\boxed{\textbf{Enter}}$ key, you may also use $\boxed{\text{Shift}}\,\boxed{\longleftarrow}$.) Now use Mathematica to evaluate the following. Remember to push the $\boxed{\textbf{Enter}}$ key after each part. Check the answers using a calculator. If the output is the same as the input, you probably have made an error in typing.

   (a) `3+6*5` (Does Mathematica follow order of operation rules?)

   (b) `274/3` (Try this again with a decimal point after either number.)

   (c) `((3.24*6.791)-14.7)/(4.5+82/3)` (Mathematica will not accept brackets or braces as computational grouping symbols. Try $[(3.24 * 6.791) - 14.7]/(4.5 + 82/3)$. Do you get the same answer?)

   (d) `2.6^2.99` (The symbol ^ is the power sign.)

   (e) `3 5` (Don't forget the space between 3 and 5.)

4. All built-in Mathematica commands or words begin with a capital letter and use "brackets" if a grouping symbol is appropriate. Type in and evaluate each of the following.

(a) `Log[12.7]` (This is the natural logarithm $\ln 12.7$.)

(b) `E^2.5` (This is $e^{2.5}$.)

(c) `Log[10,72.8]` (This is the common logarithm $\log 72.8$. Notice that the first number indicates the base.

(d) `Sqrt[64.0]` (This is $\sqrt{64}$.)

(e) `Abs[-2.5]` (This is $|-2.5|$.)

(f) `Sin[Pi/6]` (This is $\sin(\pi/6)$. Trigonometric functions are defined as a function of radians. If a degree unit is used, it must be indicated as shown next.)

(g) `Cos[30 Degree]` (This is $\cos(30°)$.)

(h) `ArcTan[1]` (This is $\tan^{-1}(1)$.)

(i) `I*I` (I is $i = \sqrt{-1}$.)

5. You can find a numerical approximation using the function `N` and `NumberForm`. Type in and evaluate each of the following.

(a) `E` (This symbol stands for the exact value of e and will only be replaced with its numerical value if asked.)

(b) `N[E]` (Approximate value of e.)

(c) `Pi` (This symbol stands for the exact value of π and will only be replaced with its numerical value if asked.)

(d) `N[Pi]` (Approximate value of π.)

(e) `Pi//N` (This is another way of asking for the numerical value of the π.)

The function `N` calculates the answer to 16 significant figures (not decimal places) but *Mathematica* only displays the default number of places. For viewing the answer to more than the default number of display places but **no more than 16** significant figures you must use the `NumberForm` command along with `N`. Try the following.

(f) `NumberForm[N[E],10]`

(g) `NumberForm[N[Pi], 16]`

You can use the command `N` by itself for calculating and displaying the answer for **more than 17** significant figures. Try the following.

(h) `N[Pi, 17]` (Did you get one more digit than the last part?)

(i) `N[Sqrt[2], 50]` (This gives the value of the irrational number $\sqrt{2}$ to 50 places.)

6. You can also input mathematics in a more standard form using palettes. A basic palette is usually present on the top right hand side of the screen. (If it is not, you can get it by clicking consecutively on the buttons Palettes, and Basic Math Assistant.) You can get a form by clicking on it in the palette. You can fill a form by clicking on each location and typing. You can also **Tab** between different locations in a form. Use the palette to type in each of the following and then evaluate each one.

   (a) $3^{5.2}$

   (b) $\sqrt{37.4}$

   (c) $\sqrt{3.1^{2.7}}$

   (d) $\frac{125.3}{72}$

7. The % key allows you to call back a previous output line. To call back the output from the previous line enter %, two lines up %%, 3 lines up %%%, etc. Evaluate the following.

   (a) `N[E^2]`

   (b) `Log[%]`   (This is the natural logarithm of the last output or $\ln(e^2)$ which, of course, is equal to 2.)

   (c) `Log[%%]`   (This is the natural logarithm of the two outputs before, which is again $\ln(e^2) = 2$.)

   (d) `Log[%]`   (What is this? Verify your answer!)

8. Mathematica can work symbolically to perform algebraic operations. Try these.

   (a) `Expand[(a+1)^5]`

   (b) `Simplify[3x^2-x-9+x^2+7x+5]`

   (c) `Factor[x^4 - 1]`

   (d) `Together[2/(x-3)-1/(x+2)]`

   (e) `Apart[5/((x-3)(x+2))]`   (The partial fractions technique command is `Apart`.)

   (f) `Cancel[(x^2-2x-3)/(x^2-9)]`

9. Replacement. A single equal sign indicates replacements. Type in the following and evaluate them.

`x = 5; y = 12; z = a + b;`

Enter all of them in just one cell; you may enter each on a separate line or all on one line. If you type in more than one on the same line, be sure to separate them by semicolons. The final semicolon in a line suppresses the final output. If you have used all the semicolons, you should not get any output! Now evaluate the following.

   (a) `x`   (This should give the assigned value of x which is 5.)

   (b) `z`   (This should give the expression that z stands for: $a + b$.)

   (c) `x y^2`   (Remember to insert the space between x and y to indicate a product.)

   (d) `Expand[z^x]`   (This is $(a + b)^5$.)

The assignments for x, y and z are permanent; this means that to reuse these variables for something else requires that they first be "cleared". Type in the following and evaluate it.

   (e) `Clear[x,y,z]`

   (f) `x`   (You should get x and not its previous value of 5. It is a good idea to clear each variable after its use, otherwise its reuse will give unintended results.)

10. In Mathematica, an equation is denoted by using two equal signs. For example, the equation `3x+2==x-5` represents the equation $3x + 2 = x - 5$. You can find exact solutions of many equations using the `Solve` command. Mathematica can solve equations of one or more variables. Should you want solutions for more than one variable, you must indicate the variables you want to solve for in a list. Lists are indicated by enclosing the entries is braces. Notice the syntax on each of the following and enter them exactly as they appear. Since x and y had numerical values previously these values must be cleared.

    (a) `Solve[3x-8==4]`   The solution is given as a replacement value for the variable ($\{x \rightarrow$answer$\}$). (If the output is an empty set, either this equation has no solution or you have forgotten to clear the value of x earlier. Check your answer.)

    (b) `Solve[a x-2==3a,x]`   (The x after the comma indicates that the equation should be solved for x and not a.)

    (c) `Solve[{x-2y==4,x-1==5y},{x,y}]`   (If you wish to solve 2 or more equations you enter them as a list.)

If an equation cannot be solved exactly, then a numerical solution may work. Below are two such examples.

    (d) `NSolve[2x-3==x^2-3x-4]`

    (e) `NSolve[{x^2+y^2==16,y==x^2-2x+2},{x,y}]`   (The solutions include points of intersections of the circle $x^2 + y^2 = 16$ and the parabola $y = x^2 - 2x + 2$.)

    (f) `FindRoot[Cos[x]==x^3,{x,1}]`   This command applies the Newton's method to the equation $\cos x = x^3$ with the starting value $x_1 = 1$.

11. The equal sign for entering a function becomes `:=` (not just `=`). The independent variables must be followed by an underline. Do not use a capital letter as the first letter in naming functions since it might get mixed up with built-in Mathematica functions. Enter the following functions and perform the indicated steps.
`f[x_]:=x^2-5; g[x_]:=Log[x]/x`

    (a) `f[3]`

    (b) `g[f[x]]`

    (c) `g[f[3]]`

    (d) Obtain a numerical approximation for part c; (`N[g[f[3]]]`).

    (e) `Solve[f[x]==4]`   This solves the equation $x^2 - 5 = 4$

12. You can draw a function, relation, or equation using the `Plot` command. Each expression must be followed by a list which gives the variable(s) and the domain (minimum value to maximum value). Enter the following.

    (a) `Plot[x^2-3x+4,{x,-10,10}]`

    Change the domain from -5 to 5 and then again from -100 to 100. You do not need a symmetric domain, try from 0 to 20.

    (b) Mathematica can plot parametric equations, in two or three dimensions.

    `ParametricPlot[{2 Cos[t],3 Sin[t]},{t,0,2Pi}]`   (This is the graph of the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$.)

    `ParametricPlot3D[{Cos[t],Sin[t],t},{t,0,10}]`   This is the graph of an ascending helix.

13. (Continued.)

    (c) Mathematica can plot 3-dimensional graphs of the form $z = f(x, y)$.
        `Plot3D[Sin[x^2 y],{x,-Pi,Pi},{y,-Pi,Pi}]`   This is the graph of $z = sin(x^2 y)$.

    (d) You can graph a polar equation using the `PolarPlot` command.
        `PolarPlot[Sin[2t],{t,0,2Pi}]`   This is the graph of $r = \sin(2\theta)$. (I used t was in place of θ since it was easier to type in!)

    (e) To graph two or more functions on the same coordinate system, enter them as a list.
        `Plot[{Cos[x],x^3},{x,-2Pi,2Pi}]`   (This is the graph of the two functions $y = \cos x$ and $y = x^3$ on the same coordinate system.)

    (f) You can read the coordinates of a point on a graph by first right clicking on the graph and choosing Get Coordinates (or, first, Drawing Tools) in the window that opens up. Move the cursor over the graph. The coordinates will appear on the screen.

    (f) You can read the coordinates of a point on a graph by first right clicking on the graph and choosing Get Coordinates (or, first, Drawing Tools) in the window that opens up. Now, try to read the coordinates of the point of intersection of the two curves in the last part. (It should be about 0.86.)

        We can use an x value near the solution (say 0.5) to get a more precise answer using the `FindRoot` command. Try the following.
        `FindRoot[Cos[x]==x^3,{x, 0.5}]`

13. Mathematics can evaluate limits. However, it is best to first investigate existence of the limit of the function by graphing it or considering a table of its values, before applying the `Limit` command. Let's investigate $\lim\limits_{x \to 0} \dfrac{\sin x}{x}$. Enter the following.

    (a) `Plot[Sin[x]/x, {x, -Pi, Pi}]`   This graph shows the limit exists.

    (a) `Limit[Sin[x]/x,x->0, Direction->"FromBelow"]`   (This is $\lim\limits_{x \to 0^-} \frac{\sin x}{x}$.)

    (b) `Limit[Sin[x]/x,x->0, Direction->"FromAbove"]`   (This is $\lim\limits_{x \to 0^+} \frac{\sin x}{x}$.)

    (c) `Limit[Sin[x]/x,x->0]`   (Since the left and right limits are equal, this is the correct value of $\lim\limits_{x \to 0} \frac{\sin x}{x}$.)

14. Mathematica can do symbolic differentiation.

    (a) `D[x^2 Sin[x]-3x+1,x]`   (This gives the derivative of $4 = x^2 \sin x - 3x + 1$ with respect to x.)

    (b) You can also use the prime notation.
        `f[x_]:=x^3-2x^2+5;f'[x]`   Is $f'(x) = 3x^2 - 4x$?
        `f''[x]`   (This gives the second derivative $f''(x)$.)
        `D[f[x],{x,3}]`   (This gives the third derivative $f'''(x)$).

    (c) You can also perform implicit differentiation.
        `D[x^2 y[x]^2 + x Sin[y[x]]==1,x]`   This differentiates the equation $x^2 y^2 + x \sin y = 1$ with respect to x. (`y[x]` indicates y is a function of x.)
        `Solve[%,y'[x]]`   Now, you have solved for y'.

15. (continued.)

(d) You can also find partial derivatives.

```
f[x_,y_,z_]:= x^4 z^3 y +x Sin[z+y];
D[f[x,y,z],x]
```

This is the first partial derivative of f with respect to x, this can also be done by `D[f[x,y,z],{x,1}]`.

`D[f[x,y,z],{x,2},{y,1},{z,1}]`   This is $\dfrac{\partial^3 f}{\partial z\,\partial y\,\partial^2 x}$.

15. Mathematica does symbolic integration. Mathematica evaluates definite integrals both numerically, and, also, by doing symbolic integration first. Input the following.

(a) `Integrate [x^2+1,x]`   (Mathematica gives the antiderivative with $c = 0$.)

(b) `Integrate [x^2+1,{x,-1,2}]`   (This evaluates $\int_{-1}^{2}(x^2 + 1)\,dx$.)

(c) `f[x_,y_]:=1-(x^2/4)-(y^2 / 9);`
`Integrate[f[x,y],{x,-2,2},{y,-3, 3}]`
This evaluates the iterated integral $\int_{-2}^{2}\int_{-3}^{3} f\,dx\,dy$ which gives the volume under the surface $z = 1 - \frac{x^2}{4} - \frac{y^2}{9}$ and above the xy-plane.

(d) Numerical Integration. Often we do not know the antiderivative of a function but we can approximate its definite integral numerically.

`NIntegrate[Exp[-x^2],{x,0,Infinity}]`   (This evaluates $\int_{0}^{\infty} e^{-x^2}\,dx$.)

(e) However, you can hand evaluate some integrals more efficiently than Mathematica. The easy way to evaluate $\int 3x^2(2010 - x^3)^{1999}\,dx$ is by u-substitution. Mathematica expands the integrand before finding its antiderivative!
`Integrate[3x^2 (2010-x^3)^1999, x]`

16. The `Series` command can be used to find the power series expansion of a function about a given point up to a given desired power. The `Sum` command can find the value of some sseries.

(a) `Series[E^x, {x,0,6}]`   (This is the Maclaurin series of $f(x) = e^x$ with all terms up to the sixth power stated explicitly and $O[x]^7$ stands for all higher order terms.)

(b) By adding the command `Normal` a series is truncated to a polynomial.
`Normal[Series[Cos[x], {x,0,8}]]`   (This is the 8th-degree Maclaurin polynomial of $f(x) = \cos x$.)

(c) `Series[Log[x], {x,1,5}]`   (This is the Taylor series expansion of $f(x) = \ln x$ about $x = 1$ with all term up to the fifth power stated explicitly.)

(d) Mathematica can find the sum of certain series.

`Sum[1/n^2, {n,1,Infinity}]`   (This gives the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$.)

17. Braces, { }, signal a "list" to Mathematica. A list can be sued to represent a set, an interval, a table of data, the parameters in a command, a vector, a matrix, etc. For a matrix, each row is inputted as a list in the order it appears and rows are separated by commas. If a row has only one entry, it is not necessary to use braces. Now try the following.

   (a) `u={1,-3}`   This is the vector $\vec{u} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$.

   (b) `MatrixForm[u]`   This command shows $\vec{u}$ in matrix form.

   (c) `v={{-2,5,-1}}`   This is the row vector $\vec{v} = \begin{bmatrix} -2 & 5 & -1 \end{bmatrix}$.
       `MatrixForm[v]`

   (d) `m={{1,2},{3,4},{5,6}}`   This is the matrix $m = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 4 & 6 \end{bmatrix}$.

18. You can add or subtract matrices using $+$ and $-$ signs. The symbol "·" is the multiplication sign for matrices. Input the following.

   (a) `n={{3,2,1},{6,5,4}}`   This is the matrix $n = \begin{bmatrix} 3 & 2 & 1 \\ 6 & 5 & 4 \end{bmatrix}$.

       `l={{3,-1},{-2,6},{-4,5}}`   This is the matrix $l = \begin{bmatrix} 3 & -1 \\ -2 & 6 \\ -4 & 5 \end{bmatrix}$.

   (b) `m-2*l`   This results in the matrix $m - 2l$.

   (c) `n.m`   This gives the product nm.
       `MatrixForm[n.m]`

19. Try the following.

   (a) `Dimensions[m]`   This gives the size of the matrix m. The first number is the number of rows and the second is the number of columns.

   (b) `i=IdentityMatrix[3]`   This is the 3×3 identity matrix i.

   (c) `d=DiagonalMatrix[{2,2,-3}]`   This is the 3×3 diagonal matrix d with the diagonal entries 2, 2 and -3.

   (d) `Transpose[m]`   This gives the transpose of the matrix m.
       `MatrixForm[Transpose[m]]`

20. Here we will perform more matrix operations. Input the following.

   (a) `a={{2.,3,-5},{-1,4,2},{5,7,2}}`   The decimal point after 2 is used to get Mathematica to work in decimal (not exact) calculations.

   (b) `Det[a]`   This gives the determinant of the matrix a.

   (c) `Inverse[a]`   This gives the inverse of the matrix a.
       `MatrixForm[Inverse[a]]`

20. (continued.)

   (d) `MatrixPower[a,3]`   This gives the matrix a^3.

   (e) `Eigenvalues[a]`   This gives the eigenvalues of the matrix a. This matrix has one real-valued and a pair of complex conjugate eigenvalues.

   (f) `Eigenvectors[a]`   This gives the eigenvectors of the matrix a.

   (g) `Eigensystem[a]`   This gives a list of eigenvalues followed by their corresponding eigenvactors.

21. We can solve a linear system using matrices. The system $\begin{cases} 2x + 3y - 5z = 14 \\ -x + 4y + 2z = 0 \\ 5x + 7y + 2z = -9 \end{cases}$ in matrix form is

$$\begin{bmatrix} 2 & 3 & 5 \\ -1 & 4 & 2 \\ 5 & 7 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 14 \\ 0 \\ -9 \end{bmatrix}.$$ Notice that the coefficient matrix is the matrix a used above.

   (a) `Solve[{2x+3y-5z==14, -x+4y+2z==0, 5x+7y+2z==-9},{x,y,z}]`   This gives the solution using the `Solve` command.

   (b) `b={14,0,-9}`   This is the constant matrix on the right side.

   (c) Now, our system is $a \begin{bmatrix} x \\ y \\ z \end{bmatrix} = b$ and the solution is $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = a^{-1} b$. Input the following.

      `Inverse[a].b`   Did you get the same answer as before?

22. The direction field of a differential equation of the form $\frac{dy}{dx} = f(x, y)$ is the plot of the vectors $\vec{v} = \langle 1, f(x, y) \rangle$ at each grid point (x, y) in the plane. The Mathematics command for such a plot is `VectorPlot`. Consider the differential equation $\frac{dy}{dx} = y - x^2$.

   (a) `VectorPlot[{1,y-x^2}, {x,-3,3}, {y,-4,4}]`   (This is the direction field of the given ODE for $-3 \le x \le 3$ and $-4 \le y \le 4$.)

   (b) You can change the number of vectors by specifying the number of vectors.
      `VectorPlot[{1,y-x^2}, {x,-3,3}, {y,-4,4}, VectorPoints->20]`

   (c) Notice that Mathematica draws all vectors with the same length. It uses the vector colors to indicate the length of vectors. To preserve the length of vectors, use the following command.
      `VectorPlot[{1,y-x^2},{x,-3,3},{y,-4,4},VectorScaling->Automatic,VectorPoints->20]`

   (d) Mathematica can display the vectors as part of the approximate trajectories of the solutions using the `StreamPlot` command.
      `StreamPlot[{1,y-x^2}, {x,-3,3}, {y,-4,4}]`

   (e) The same commands work systems of ODE's. The vector field for the system $\frac{dy}{dx} = f(x, y)$, $\frac{dy}{dx} = g(x, y)$ is the graph of vectors of the form $\vec{v} = \langle f(x, y), g(x, y) \rangle$. Consider the system $\frac{dy}{dx} = -x + x^3$, $\frac{dy}{dx} = -2y$. Try each of the following.
      `VectorPlot[{-x+x^3, -2y}/(0.1+(-x+x^3)^2+(-2y)^2)^(1/2), {x,-3,3}, {y,-4,4}]`
      `StreamPlot[{-x+x^3, -2y}, {x,-3,3}, {y,-4,4}]`

23. Using the `DSolve` and `NDSolve` commands, we can find solutions to many differential equations.

    (a) The syntax for the general solution of $y' = f(x, y)$ is `DSolve[y'[x]==`$f(x, y)$`, y[x], x]`. Consider the equation $\frac{dy}{dx} = y - x^2$.
        `DSolve[y'[x]==y[x]-x^2, y[x], x]`   (`C[1]` in the answer is the constant c_1.)

    (b) The syntax for the solution of the IVP $y' = f(x, y)$, $y(x_0) = y_0$ is `DSolve[{y'[x]==`$f(x, y)$`,`
        `y[`x_0`]==`y_0`}, y[x], x]`. Consider the IVP $\frac{dy}{dx} = y - x^2$, $y(0) = 2$.
        `DSolve[{y'[x]==y[x]-x^2,y[0]==2}, y[x], x]`

    (c) In a similar fashion, you can solve a system of equations.
        `DSolve[{x'[t]==2x[t]+y[t],y'[t]==x[t]-2y[t]}, {x[t],y[t]}, t]`   (This gives the general
        solution of the system $\frac{dx}{dt} = 2x + y$, $\frac{dy}{dt} = x - 2y$.)
        `DSolve[{x'[t]==2x[t]+y[t],y'[t]==x[t]-2y[t], x[0]==2, y[0]==1}, {x[t],y[t]}, t]`   (This
        gives the solution of the IVP system $\frac{dx}{dt} = 2x + y$, $\frac{dy}{dt} = x - 2y$, $x(0) = 2$, $y(0) = 1$.)

    (d) Sometimes the ODE can only be solved numerically or you may just want to graph the solution. In that case the command `NDSolve` is used and its output is graphed. The syntax for this command is `NDSolve[{y'[x]==`$f(x, y)$`, y[`x_0`]==`y_0`}, y[x], {x, `x` min, `x` max}]`.
        `NDSolve[{y'[x]==y[x]-x^2,y[0]==2}, y[x], {x, -3,3}]`   (Now, Mathematics has the numerical solution, a collection of points on the solution, for $-3 \le x \le 3$.) To graph it, do the following.
        `Plot[Evaluate[y[x] /. %], {x, -3,3}]`   (`/.` means replace all and `y[x] /. %` means replace `y[x]` with the last output.)

    (e) To graph the solution of the IVP system $\frac{dx}{dt} = -x + x^3$, $\frac{dy}{dt} = -2y$, $x(0) = 0.5$, $y(0) = 1$, do the following.
        `NDSolve[{x'[t]==2x[t]+y[t],y'[t]==x[t]-2y[t],x[0]==2,y[0]==1},{x[t],y[t]},{t,-2,2}]`
        `ParametricPlot[Evaluate[{x[t], y[t]} /. %], {t, -2, 2}]`

    (f) We can graph the direction field and a solution together. The following is the direction field for $\frac{dy}{dx} = y - x^2$ and graph of the solution of the IVP for the same ODE with the initial condition $y(0) = 2$.
        `graph1=VectorPlot[{1, y - x^2}, {x, -3, 3}, {y, -4, 4}]`
        `NDSolve[{y'[x]==y[x]-x^2, y[0]==2}, y[x], {x, -3, 3}]`
        `graph2=Plot[Evaluate[y[x] /. %], {x, -3, 3}]`
        `Show[graph1, graph2]`

    (g) To combine a direction field with graphs of solutions of several IVP's with for the same ODE, use the `Table` command. The following is as in the last part except that initial conditions are $y(0) = n$ for $n = -3, -2, -1, 0, 1, 2, 3$.
        `graph1=VectorPlot[{1, y - x^2}, {x, -3, 3}, {y, -4, 4}]`
        `Table[NDSolve[{y'[x]==y[x]-x^2, y[0]==n}, y[x], {x,-3,3}],{n,-3,3,1}]`
        `graph3=Plot[Evaluate[y[x] /. %], {x,-3,3}]`
        `Show[graph1, graph3]`

24. It is essential, as a courtesy to future users, and continued problem-free usage, to leave the computers as you find them. When you are done, close the application and/or log off properly.